Bio 352 Quantitative Questions

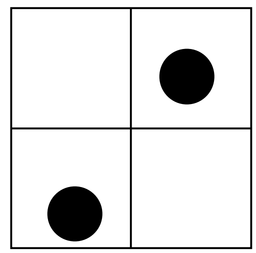
1) Scavenging female crabs may encounter one another at a fish carcass. In the ensuing contest for this potentially rich food resource, a crab may either immediately attack its opponent by lunging with its claws or, alternatively, it may display threateningly at the other crab, but not attack. When both crabs display at each other, half the time they end up sharing the carcass equally, while the other half of the time, one simply leaves and the other gets the entire carcass to itself. A displaying crab always gives way to an attacking crab before a fight begins; however, if both crabs attack, there is a 75 % chance that a crab will lose a claw in the ensuing struggle, irrespective of who eventually wins. The winning crab gains exclusive access to the carcass while the loser gets nothing. Going into such a contest, crabs have an equal probability of winning.

Based on these probabilities and presuming that access to the entire fish carcass provides a crab with, on average, enough energy to produce 800 eggs, while the energy needed to regenerate a new claw will reduce egg production by 600 eggs, describe what behavioral strategies are expected to evolve in terms of displaying and attacking. If a polymorphic strategy is predicted, calculate the expected frequency of attacking. Show all your work.

This is a classic 2x2 symmetrical game theory problem with the following values:

	Fight	***Display***
Fight	P_{11}	P_{12}
Display	P_{21}	P_{22}

-50	***800***
0	***400***



Payoffs (expected total number of eggs produced):

P_{11} (both fight) = (prob of winning) x (payoff of winning - cost of fighting) + (prob of losing) x (payoff of losing - cost of fighting)

Where: payoff to winner = 800; prob of winning = 0.5; payoff to loser = 0; prob of losing =0.5

Cost of fighting = 0.75 x - 600 = - 450 (reduction in egg production)

So P_{11} = 0.5(800 - 450) + 0.5(0 – 450) = 175 – 225 = -50

P_{12} (one fights, opponent displays) = prob of winning x ben of winning = 1 x 800 = 800
P_{21} (one displays, opponent fights) = prob of winning x ben of winning = 0 x 800 = 0
P_{22} (both display) = (prob of sharing x ben of sharing) + prob of getting all x ben of getting all) + prob of leaving x ben of leaving) = .5(400) + .25(800) + .25(0) = 200 + 200 = 400

Plugging these payoffs into the matrix we find a mixed stable strategy for fighting and displaying. The frequency of fighting can be calculated from the equation: $f = (P_{12}-P_{22})/((P_{12}-P_{22})+(P_{21}-P_{11}))$
For the payoffs given, f = (800-400)/((800-400)+(0-(-50))=400/450=0.889 or 8/9.

How would your answer change if the risk of losing a claw during a fight drops to 20%?

If the risk of losing a claw drops to 20% then the cost of fighting is now .2(-600) = -120
Thus, P_{11} becomes .5(800 - 120) + .5(0 – 120) = 340 – 60 = 280 and a pure strategy for "fight" is predicted.

Bio 352 practice questions

2) Gulls feeding on fish in the waters surrounding their nesting colonies perform one of two types of foraging strategies. Some gulls ("foragers") catch their own fish, while others ("chasers") give up some of their own foraging to chase after other gulls. Chasers harass other birds until they drop their catch, which is then eaten by the chaser.

Suppose "foragers" can, on average, catch 400 fish per day if they are not chased. Chasers that find "foragers" will steal 75% of this catch, however, "chasers" will also lose 120 fish per day because of the reduced time they spend foraging for their own fish. If a "chaser" tries to chase another "chaser", a protracted chase ensues, since neither bird has a fish to lose. When this happens, chasers can expect to lose an ***additional*** 200 fish per day. If gulls are otherwise equal to one another and cannot tell whether another gull is a "forager" or a "chaser", predict how many gulls you would expect to see employing a chasing strategy in a colony of 1000 birds. Would your answer change if fish densities increased and the daily catch grew to 4000 fish but otherwise, expected losses did not change? Show all your work.

This is a classic 2x2 symmetrical game theory problem:

	Forage	***Chase***
Forage	F	$F-B$
Chase	$F+B-C_1$	$F+C_1-C_2$

Let ***F = Expected gain of foraging*** ***B = the gain from chasing***
C_1 = loss from chasing a forager ***C_2 = loss from chasing a chaser***

Where $F = 400, B = 300, C_1 = 120, C_2 = 200$

Payoffs would be calculated as follows:

$P_{11} = F = 400$
$P_{12} = F - B = 400 - 300 = 100$
$P_{21} = F + B - C = 400 + 300 - 120 = 580$
$P_{22} = F - C1 - C2 = 400 - 120 - 200 = 80$

400	100
580	80

Plugging these values into the Payoff matrix reveals a mixed, stable ESS

The frequency of playing a "forage" strategy at equilibrium can be calculated as:

$f = (P_{12}-P_{22})/((P_{12}-P_{22})+(P_{21}-P_{11}))$
$= (100-80)/((100-80)+(580-400))=20/200=0.1$ ***or*** $1/10$.

The question asked about the predicted number of birds chasing. This is given my multiplying the frequency of chasing (1-f) by the size of the population (1000). This value = (1 - 0.1) * 1000 = 0.9 * 100 = 900 birds.

If fish densities increased forager success tenfold, to 4000, then the Payoff matrix would be:

4000	1000
6880	3680

This is based on the following calculations:

$P_{11} = F = 4000$
$P_{12} = F - B = 4000 - 3000 = 1000$
$P_{21} = F + B - C = 4000 + 3000 - 120 = 6{,}880$
$P_{22} = F - C1 - C2 = 4000 - 120 - 200 = 3{,}680$

Now a Pure ESS for "Chase" is predicted… all birds should chase as a strategy to gain food.

Bio 352 practice questions

3) A female songbird that lays a single clutch of 4 eggs each breeding season can establish her nest in one of two habitat types. In the first habitat, the insects she will use to feed her offspring are larger and more abundant, such that there is a higher probability of successfully feeding more chicks (see table). There is also, however, a higher risk of nest predation, with a 40% chance that the entire clutch will be wiped out. In the other habitat, insect density is lower, with a lower expected probability of successfully feeding the chicks (see table), but the likelihood of the clutch being eaten by a predator drops to 5%. Based on this information, in which habitat should the female establish her nest if she is attempting to maximize chick survivorship each breeding season? Show all your work

	Probability of occurrence	
# of chicks successfully fed	**Habitat I**	**Habitat II**
4	0.6	0.1
3	0.3	0.3
2	0.1	0.5
1	0.0	0.1
0	0.0	0.0

To answer this question, you must first calculate the expected clutch success, irrespective of predation on the clutch. This would be the sum of the products of each clutch size multiplied by its probability of occurrence.

For Habitat I: 4(0.6) + 3(0.3) + 2(0.1) + 1(0) = 2.4 + 0.9 + 0.2 = 3.5

For Habitat II: 4(0.1) + 3(0.3) + 2(0.5) + 1(0.1) = 0.4 + 0.9 + 1.0 + 0.1 = 2.4

Next these values must be multiplied by the probability of <u>not</u> being eaten by predator. If p is the probability of predation then (1 – p) is the probability of no predation.

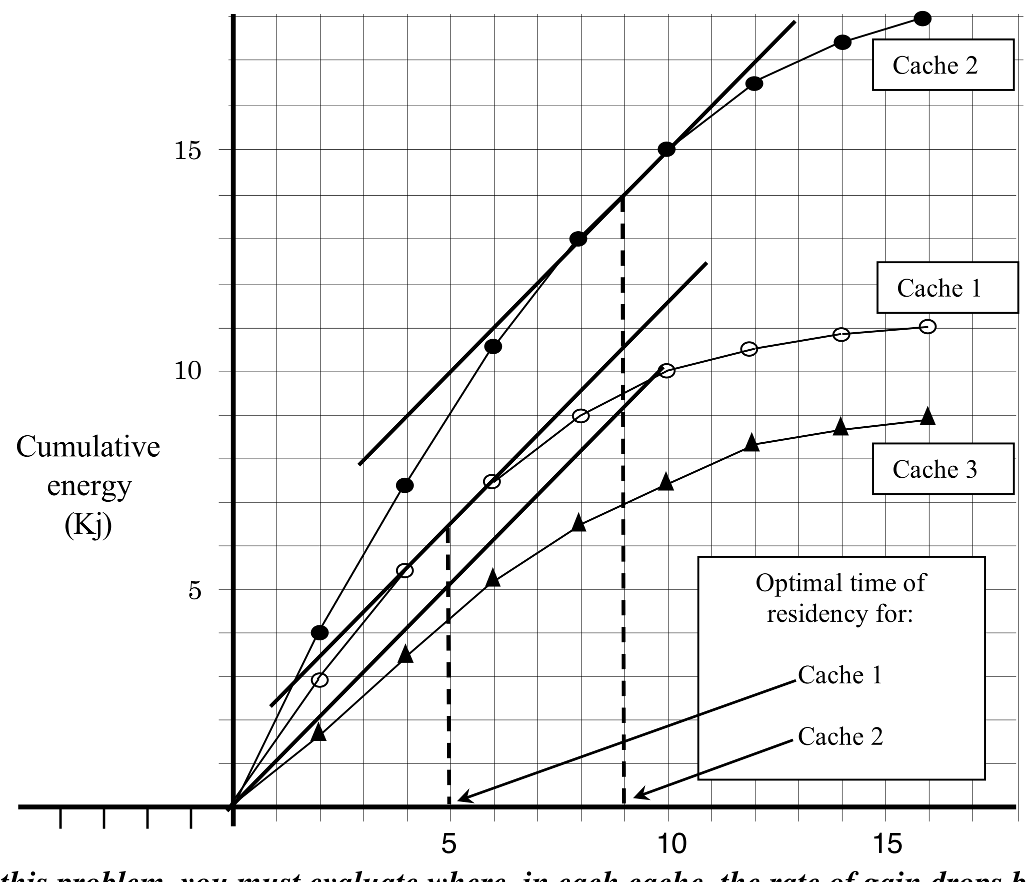
For Habitat I, p = 0.4, and 1-p = 0.6, 3.5(0.6) = 2.1 This is the expected success in Habitat I.

For Habitat II, p = 0.05, and 1-p = 0.95, 2.4(0.95) = 2.28 This is the expected success in Habitat II.

Thus, Habitat II would be a better choice.

4) During the fall, field mice gather seeds and store them in various spots within their home range. They exploit these caches during the winter. Because seed caches vary in quality, mice receive different rates of food intake as a function of time within a given cache. Behavioral ecologists have identified three types of seed cache that generate three different *cumulative* rates of energetic return (in Kilojoules, see table). Assuming that: 1) seed caches are randomly distributed within the homerange, with travel time between caches averaging 4 minutes; and 2) that average rate of food intake, including travel time, is 1 Kj/min; estimate the optimal time a mouse should spend in a each type of cache using the available data and the axes below. Show all your work and provide a complete explanation of any graphical approaches you might use.

Time (min)	Cache 1	Cache 2	Cache 3
2	3.00	4.00	1.75
4	5.50	7.50	3.50
6	7.50	10.50	5.25
8	9.00	13.00	6.50
10	10.00	15.00	7.50
12	10.50	16.50	8.25
14	10.75	17.50	8.75
16	11.00	18.00	9.00



To solve this problem, you must evaluate where, in each cache, the rate of gain drops below the average (1 Kj/min). That is the time to leave. You can do this graphically, if you draw carefully, by finding the intersection of each curve with a tangent line of slope = 1. Even without a plot, you can see that Cache 3 never has a gain rate (slope) of greater than 1.0, so the mouse should never enter those caches. Cache 2 yields the highest gain, with a slope of 1.0 hitting the curve between minute 8 and minute 10 (i.e., the gain during that time is 13 - 15 = 2 and the time interval is 2 minutes; 2/2 – 1.0). This is when those caches should be abandoned. They should stay in cache 1 for between 4 and 6 minutes using similar reasoning